PREDICTION METHODS FOR PARAMETRIC ROLLING
UNDER DRIFTING CONDITION AND THEIR VALIDATION
-FINAL REPORT OF SCAPE COMMITTEE (PART 3) –
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ABSTRACT
In this paper, experimental and theoretical research works on heavy parametric rolling in beam waves which have been done for these three years by the authors are reviewed, and the recommendations to IMO on the basis of the results of these works are made.

KEY WORDS: Parametric rolling, Beam waves, Large passenger ship, Dead ship condition, Roll damping, Heave resonance, Heading angle.

NOMENCLATURE

\( \Phi \) : roll angle
\( \Phi_0 \) : roll amplitude
\( \Phi_{\text{max}} \) : maximum roll amplitude at the peak of parametric rolling
\( H_w \) : wave height
\( \zeta \) : wave amplitude (\( =H_w / 2 \))
\( Z_0 \) : heave amplitude
\( T_R \) : roll natural period
\( T_{\text{ob}} \) : heave natural period
\( T_{\text{s}} \) : roll period
\( T_h \) : heave period
\( T_e \) : encounter wave period (\( =T_h \))
\( T_i \) : incident wave period
\( T_{\text{rp}} \) : encounter wave period at the peak of parametric rolling

INTRODUCTION
As well known, occurrence of parametric rolling of ships without forward speed under drift condition in beam waves has been pointed out by many researchers, as follows. Paulling and Rosenberg (1960) pointed out that Froude (1863) observed that ships have unwanted roll characteristics if the natural frequencies in heave and roll are in the ratio of 2:1 in beam waves, and they investigated unstable roll motion produced by nonlinear coupling from heave motion theoretically and experimentally. Tasai (1965) noted that small unstable roll motion caused by periodic roll restoring variation in regular beam waves. Tamiya (1969) measured unstable roll motion and capsizing of a cylindrical model in beam waves and wind, and pointed out that the phenomena can be explained by a Mathieu equation. Sadakane (1978) studied on the effect of fluctuation of the apparent weight of a ship in regular heavy waves on roll motion, found that the heavy rolling has harmonic and subharmonic oscillations by experiments, and noted that the phenomena can be explained by a Mathieu-type instability. Blocki (1980) reported experimental results of capsizing of a cylindrical model of a fishing vessel due to parametric rolling in regular beam waves, and calculated capsizing probability caused by parametric resonance. Boroday and Morenschildt (1986) carried out an experimental investigation of the condition giving rise to the cylindrical ship models with and without flare parametric rolling in regular and irregular beam waves, and found large parametric rolling occurs when the ship has large flare. Umeda et al. (2002) experimentally showed that parametric rolling for a fishing vessel under drifting condition without forward velocity occurs in head waves, and Munif and Umeda (2006) concluded that coupling with pitch is essential to numerically explain this type of parametric rolling.

Recently, Ikeda et al. (2005A) experimentally showed that heavy roll motion with much larger angle than that in 1st harmonic resonance appears for a modern large passenger ship with flat stern and large bow flare in beam waves due to parametric rolling. The parametric rolling in beam waves disappears when the roll damping is large enough. This fact suggests that the roll damping of such ships should be designed not to occur heavy parametric rolling in beam waves as well as to meet the intact stability criteria.
Following the paper, the authors have been carrying out experimental and theoretical investigations on the parametric roll in beam waves.

EXPERIMENTAL RESULTS OF PARAMETRIC ROLLING IN BEAM WAVES

Measurements of roll motions of a model ship were carried out in the towing tank of Osaka Prefecture University. The model ship used in the experiments is an 110,000 GT passenger ship designed for international cooperative research projects on damage stability for contributing to IMO regulatory works. The body plan and the principal particulars are shown in Fig.1 and Table 1, respectively.

![Fig. 1 Body plan of the ship.](image)

**Table 1 Principle particulars.**

<table>
<thead>
<tr>
<th></th>
<th>Full Scale</th>
<th>Model</th>
</tr>
</thead>
<tbody>
<tr>
<td>Scale</td>
<td>1/1</td>
<td>1/125.32</td>
</tr>
<tr>
<td>$L_{OA}$</td>
<td>290 m</td>
<td>2.200 m</td>
</tr>
<tr>
<td>$L_{PP}$</td>
<td>242.24 m</td>
<td>1.933 m</td>
</tr>
<tr>
<td>$B_M$</td>
<td>36 m</td>
<td>0.287 m</td>
</tr>
<tr>
<td>$D$</td>
<td>8.4 m</td>
<td>0.067 m</td>
</tr>
<tr>
<td>Displacement</td>
<td>53,010 ton</td>
<td>26.98 kg</td>
</tr>
<tr>
<td>$GM$</td>
<td>1.579 m</td>
<td>0.0126 m</td>
</tr>
<tr>
<td>$T_\text{nr}$</td>
<td>23 sec</td>
<td>2.05 sec</td>
</tr>
<tr>
<td>Bilge keel : width</td>
<td>1.1 m</td>
<td>0.0088 m</td>
</tr>
<tr>
<td>Bilge keel : location</td>
<td>s.s.3.0 – 5.0, s.s.5.25 – 6.0</td>
<td></td>
</tr>
</tbody>
</table>

Fig. 2 shows measured roll amplitude of the model ship without bilge keels in beam waves with regular wave height of 0.04 m. The peak amplitude of roll resonance at the natural roll period ($T_\text{nr}$=2.04 sec) reaches 7 degrees, but the roll amplitude at encounter period of nearly half of the natural roll period ($T_e/2$=0.85 sec) reaches 25 degrees. As shown in Fig. 3, the period of the large roll motion is twice of the encounter wave period and almost the same as the natural roll period. It should be noted that the wave encounter period where the large roll appears is near the natural heave period of the ship, too (Ikeda et al., 2005B).

![Fig. 2 Measured roll amplitude for naked hull in regular beam waves of 0.04 m wave height.](image)

It is experimentally found that the heavy rolling with twice period of the wave encounter period strongly depends on wave height. The maximum amplitude of the heavy roll at each wave height is shown in Fig. 4. The results demonstrate that the heavy rolling appears just over 30 mm of wave height, rapidly increases with wave height, and reaches the maximum amplitude that is about 27 degrees. It should be noted that the roll amplitude does not proportionately increase with increasing wave height but seems to saturate to the maximum one (Ikeda et al., 2006).

![Fig. 3 Comparison between measured roll frequency and encounter frequency.](image)

![Fig. 4 Effect of wave height on maximum amplitude of parametric rolling of the ship in regular beam waves.](image)

EXPERIMENTAL RESULTS OF PARAMETRIC ROLLING IN ALL HEADING ANGLES UNDER DRIFTING CONDITION

To know the characteristics of the parametric rolling in beam waves, some investigations on parametric rolling in other heading angles to waves are carried out.

In Fig. 5, difference of wave periods when parametric rolling appears in beam and head waves is shown. The results suggest that parametric rolling appears in different regions of wave period in beam and head waves. This may be because of differences of drift speed and amplitude of stability variation in beam and head waves (Fujiwara et al., 2006B).
Fig. 5  Difference of wave periods for which parametric rolling appears in beam and head waves.

To investigate the effect of heading angle on the parametric rolling of the dead ship, the model are released in head sea condition \( (\chi=180^\circ) \) or following sea condition \( (\chi=0^\circ) \) in regular waves. The time histories of the heave, pitch, roll and yaw motions are measured and shown in Figs. 6 and 7. From the time histories of the yaw angles, we can see that, in both cases, the heading angle of the ship slowly changes to beam sea condition. The results demonstrate that the parametric rolling occurs in wide heading angles as well as in beam waves.

Using the results of the measurements shown in Figs. 6 and 7, the roll amplitude of the parametric rolling for each heading angle is plotted in a polar diagram as shown in Fig. 8. The results show that amplitudes of the parametric rolling are significant in following and head waves as well as in beam waves when no bilge keel is attached. In Fig. 9, the effect of heading angle on parametric rolling of the ship with the designed bilge keels (BK1+BK2 that is shown in Table 2) is shown in a polar diagram. We can see that in head and following waves parametric rolling occurs even if size of bilge keels is large enough to erase it in beam waves (Munif et al., 2006).

Fig. 6  Time histories of motions of the ship without bilge keel released from head sea condition in regular waves at \( T_w=0.95 \text{ sec} \) and \( H_w=0.04 \text{ m} \).

Fig. 7  Time histories of motions of the ship without bilge keel released from following wave condition in regular waves at \( T_w=0.95 \text{ sec} \) and \( H_w=0.04 \text{ m} \).

Fig. 8  Effect of wave direction on amplitude of parametric rolling of the ship without bilge keel in regular waves.
EFFECTS OF ROLL DAMPING

As shown in the previous chapter, parametric rolling in beam waves significantly depends on the roll damping of ships. For the ship used in our experiments, the designed bilge keels can erase the parametric rolling in beam waves as shown in Fig. 9.

By changing the roll damping systematically, the effect of the roll damping on the rolling was investigated. The length and attached position of various bilge keels are shown in Table 2.

Table 2 Size and location of bilge keels.

<table>
<thead>
<tr>
<th>BK location</th>
<th>s.s. no.</th>
<th>Area of Bilge Keel / Area of Designed Bilge Keel</th>
</tr>
</thead>
<tbody>
<tr>
<td>BK 1</td>
<td>5.25 - 6.00</td>
<td>150 mm</td>
</tr>
<tr>
<td>Mid 1</td>
<td>5.00 - 6.00</td>
<td>200 mm</td>
</tr>
<tr>
<td>Mid 2</td>
<td>4.75 - 6.00</td>
<td>250 mm</td>
</tr>
<tr>
<td>Mid 3</td>
<td>4.70 - 6.00</td>
<td>260 mm</td>
</tr>
<tr>
<td>Mid 4</td>
<td>4.60 - 6.00</td>
<td>280 mm</td>
</tr>
<tr>
<td>Mid 5</td>
<td>4.50 - 6.00</td>
<td>300 mm</td>
</tr>
<tr>
<td>Mid 6</td>
<td>4.25 - 6.00</td>
<td>350 mm</td>
</tr>
<tr>
<td>BK 2</td>
<td>3.00 - 5.00</td>
<td>400 mm</td>
</tr>
<tr>
<td>BK1 + BK2</td>
<td>5.25 - 6.00</td>
<td>150 mm</td>
</tr>
<tr>
<td></td>
<td>3.00 - 5.00</td>
<td>400 mm</td>
</tr>
</tbody>
</table>

In Fig. 10, the maximum roll amplitudes of parametric rolling for different length of bilge keels in regular beam waves of 0.04 m height are shown. The results demonstrate that the peaks of parametric rolling rapidly decreases with increasing area of bilge keels, or roll damping, and disappears at half area of the designed bilge keels.

In order to clarify the effect of the roll damping on the critical wave height to occur parametric rolling and on the its maximum amplitude of the ship in regular beam wave, measurements of ship motions of the model in the cases with BK 1, Mid 2-BK, Mid 3-BK and without bilge keel are carried out in regular beam waves with various wave heights. The results of the experiments are shown in Fig. 11. These results demonstrate that the minimum wave height at which parametric rolling appears increases with increasing the roll damping, and that the roll amplitudes saturate with increasing wave height, and the saturated roll amplitudes also depend on the roll damping.

In Fig. 12, measured results of the roll damping for each bilge keel shown in Table 2 in terms of $N$ coefficient are shown. $N$ coefficients of the ship increase with increasing area of bilge keels. It should be noted that $N$ coefficient for a bare hull is 0.018 as shown in Fig. 12. The value is not so small but near 0.02 which is used for a ship with bilge keels in the Japanese Stability Code for passenger ships. The fact may suggest that careful design for bilge keels should be done to secure for escaping from large parametric rolling in beam waves (Fujiwara et al., 2006).
EFFECTS OF HEAVE RESONANCE

In Fig. 2, it can be seen that large parametric rolling appears at encounter period between half of natural roll period and natural heave period. The results may suggest that heave resonance may be one of causes of parametric rolling in beam waves. In order to investigate this hypothesis, measurements of roll motions of the model ship are carried out for various degree of coincidence between half of natural roll period and natural heave period, $\frac{T_{nr}}{2T_{nh}}$. Natural roll periods of the ship are systematically changed by changing moment of inertia, as shown in Table 3. This means that the centre of gravity of the ship keeps at original location.

Table 3  Conditions of model experiments

<table>
<thead>
<tr>
<th>$T_{nr}$ (sec)</th>
<th>$\frac{T_{nr}}{2T_{nh}}$</th>
<th>$T_n$ (sec)</th>
<th>$H_w$ (m)</th>
</tr>
</thead>
<tbody>
<tr>
<td>1.18</td>
<td>0.787</td>
<td>0.40 ~ 1.18</td>
<td></td>
</tr>
<tr>
<td>1.40</td>
<td>0.921</td>
<td>0.45 ~ 1.40</td>
<td></td>
</tr>
<tr>
<td>1.65</td>
<td>1.18</td>
<td>0.50 ~ 1.65</td>
<td></td>
</tr>
<tr>
<td>2.04</td>
<td>1.38</td>
<td>0.60 ~ 2.30</td>
<td></td>
</tr>
<tr>
<td>2.39</td>
<td>1.63</td>
<td>0.70 ~ 2.40</td>
<td></td>
</tr>
<tr>
<td>2.92</td>
<td>1.92</td>
<td>1.00 ~ 2.92</td>
<td></td>
</tr>
<tr>
<td>3.33</td>
<td>2.25</td>
<td>1.40 ~ 1.70</td>
<td></td>
</tr>
</tbody>
</table>

Effects of the ratio of half of natural roll period to natural heave period, $\frac{T_{nr}}{2T_{nh}}$ on the peak of roll amplitude of the parametric rolling are shown in Fig. 13. Intuitively, maximum parametric rolling must appear at when $\frac{T_{nr}}{2T_{nh}}$ is 1, if heave resonance causes the parametric rolling in beam waves. The results shown in Fig. 13, however, demonstrate that maximum parametric rolling occurs at when $\frac{T_{nr}}{2T_{nh}}$ is 1.38. This fact may suggest that heave resonance only partly causes parametric rolling in beam waves.

Measured amplitudes of heave motions and its phase angles from incident waves are shown with calculated results by a strip method (OSM) in Figs. 14-16. The measured heave amplitudes in the both cases shown in Figs. 14 and 15 are very similar to each other, and do not have any high peaks. The phase angle shown in Fig. 16 rapidly changes by $2\pi$ near heave resonant period (Fujiwara et al., 2007).

An example of time histories of roll, heave and incident waves are shown in Fig. 17. Using these measured data, relative heave motion to wave surface, or time variation of the draft of the ship, can be calculated. The results of the calculations are shown in Fig. 18. We can see that the time-variation of the draft is large at the heave resonant period, and gradually decreases with far from it. The time variation of the draft in beam waves may cause change of stability.

Two examples of the time-variation of the draft of the ship are shown with the amplitudes of parametric rolling in beam waves in Figs. 19 and 20. These figures demonstrate that the parametric rolling occur when the relative heave motion becomes significant. Furthermore, it occurs even if the time-variation of the draft comparatively small (Ikeda et al., 2007).

Fig. 13  Effect of ratio of half of natural roll period to natural heave period on maximum roll amplitudes at peaks of parametric rolling in beam waves.

Fig. 14  Heave response curve in the cases when parametric rolling occurs.

Fig. 15  Heave response curve in the cases when parametric rolling does not occurs.

Fig. 16  Phase angle of heave motion from incident waves in the cases when parametric rolling occurs.
THEORETICAL MODELS OF PARAMETRIC ROLLING IN BEAM WAVES

Using a nonlinear 1 DOF model of roll equation, a simulation of the parametric rolling in beam waves is carried out. Nonlinear damping and nonlinear restoring moments are considered in this model as shown in Eq. (1).

\[
\ddot{\phi} + 2\alpha\dot{\phi} + \beta\phi + \gamma\phi^3 + \omega_e^2 \left(1 + \Delta GM/2GM\right) \cos \omega_t t \phi + \\
\omega_e^2 \left(\frac{a_1}{GM} + \Delta \alpha_1\right) \phi^3 + \omega_e^2 \frac{a_3}{GM} \phi^5 + \omega_e^2 \frac{a_5}{GM} \phi^7 = \zeta_{rk} \omega_e^2 \sin \omega t
\]  

for the polynomial equation up to 7th order of restoring moment. \(\alpha\), \(\beta\) and \(\gamma\) are the roll damping coefficients and \(a_1\), \(a_3\) and \(a_5\) are the nonlinear coefficient of restoring moment in still water. These coefficients can be determined from the fitting of the GZ curves in still water. \(\Delta GM\), \(\Delta \alpha_1\), \(\Delta \alpha_3\) and \(\Delta \alpha_5\) are the coefficients of the restoring moment in waves, and they are the function of wave steepness and relative position of ship in waves. These coefficients are calculated by Froude-Krylov assumption.

The numerical results, as shown in Fig. 21, demonstrate the same tendency with the experimental results. However the numerical results underestimate the experimental results. This could be due to change of metacentric height in the numerical results much lower than the real change of metacentric height in the experiments, or this could be due to the change rolling period, which was identified in the model experiments. But critical wave steepness can be estimated approximately (Munif et al., 2005).

RECOMMENDATION FOR IMO CRITERIA

For a ship having large slope of side-walls and flat-shallow stern shape with the natural roll period close to twice as the natural heave period, it is recommended to examine the possibility of parametric rolling not only in longitudinal waves but also in beam waves. An appropriate size of bilge keels should be designed to erase the parametric rolling in beam waves to guarantee the safety in dead ship condition.

CONCLUSIONS

The activity of the SCAPE committee for parametric rolling in beam waves under drifting condition provides the following conclusions:

1. Parametric rolling in beam waves can be significant for
a large passenger ship if her bilge keels are removed.

2. If she is equipped with the designed bilge keels, the parametric rolling in beam waves disappears but that in longitudinal waves does not.

3. When the bilge keel size increases, the critical wave height of parametric rolling in beam waves and its magnitude increases.

4. When the relative heave motion in beam waves becomes significant and the roll damping is not sufficiently large, parametric rolling in beam waves could occur. The ratio of natural periods of heave and roll on its own does not explain the amplitude of parametric rolling in beam waves.

5. The critical wave height for parametric rolling in beam waves can be roughly estimated with a numerical simulation with roll restoring variation in time.

REFERENCES


Froude, W., 1863, “Remarks on Mr. Scott-Russell’s Paper on Rolling”, IA4


