Ultimate Strength Analysis of Stiffened Panels Subjected to Biaxial Thrust Using JTP and JBP Methods

1. Introduction

A series of ultimate strength analyses of stiffened panels subjected to biaxial thrust is performed using JTP method (PULS), JBP method (GL-formulae) and FEM, and the results are compared. The nonlinear shell finite element code, ULSAS, is used for the FE analysis. The accuracy of ULSAS has been verified by several benchmark studies including those performed by ISSC [1]. The comparison between PULS and FEM is first presented, and then that among PULS, GL-formulae and FEM.

2. Model for FE analysis

Ultimate strength of a continuous stiffened panel subjected to biaxial thrust as illustrated in Fig.1 is calculated by FEM. A triple span-double bay model shown by the shaded area in Fig.1 is employed. Symmetric boundary conditions are imposed at two longitudinal edges, while periodically symmetric boundary conditions are imposed at two transverse edges so as to consider the buckling mode of local plate panels and stiffeners having either even or odd number of longitudinal half waves. The transverse members are not modeled, but the panel is simply supported along those members. Considering the continuity of plating, the in-plane displacement at four edges in their perpendicular direction is assumed to be uniform. A bilinear degenerated shell element with four nodes is used. One stiffener space is divided by eight elements, and the similar mesh fineness is applied in the plate panel lengthwise and in the stiffener web depthwise. The stiffener flange is divided by six elements in its width direction.

The stiffener space, \( b \), is set to 800mm, while the aspect ratio, \( a/b \), the plate thickness, \( t \), and the stiffener type and dimensions are systematically changed. Young's modulus, Poisson's ratio and yield strength are taken as 205.8GPa, 0.3 and 314MPa, respectively. The strain-hardening effects are not taken into account.

In GL-formulae, the initial panel and stiffener deflections are not explicitly taken into account in the calculation of the ultimate strength.

In PULS, the initial deflection of a continuous stiffened panel is expressed by the sum of local and global modes. The local mode is represented by the elastic buckling mode of a panel-stiffener combination supported along the stiffener lines. The same number of half waves is assumed for local panel and stiffener. The maximum amplitude of the initial deflection is taken as 1/200 of a stiffener space as a default value. The global initial

![Fig.1 A continuous stiffened panel](image1)

![Fig.2 Initial deflection](image2)
deflection mode is represented by the elastic buckling mode of an equivalent orthotropic plate model. The maximum deflection is taken as 1/1000 of a stiffener span as a default value.

In the present FE analysis, the initial deflections of local plate panels and stiffeners are assumed separately and then superimposed. For the plate panels, two initial deflection modes are considered. One is the elastic buckling mode with \( m \) half waves, Fig.2 (a), expressed by

\[
W_{pp} = \delta_{pp} \sin \frac{m \pi x}{a} \sin \frac{\pi y}{b},
\]

and the other a hungry-horse mode expressed by

\[
W_{pp} = \sum \delta_{pp} \sin \frac{m \pi x}{a} \sin \frac{\pi y}{b}.
\]

In the latter case, the initial deflections in the same lateral direction are assumed in local plate panels with the 10% difference in the maximum deflection between adjacent bays and spans. The relative amplitude of deflection components, \( \delta_{pp} \), in Eq. (2) are determined based on the measurement of initial panel deflection in actual ships [2]. The maximum amplitude of the initial deflection is taken as 1/200 of a stiffener space, i.e. a default value in PULS. According to Smith et al [3], the average level of the maximum initial panel deflection, \( \delta_{p0} \), is given by

\[
\delta_{p0} = 0.1 \beta^{1/2}, \quad \beta = \frac{b}{t} \sqrt{\frac{\sigma_{y}}{E}},
\]

where \( \beta \) is a reduced slenderness of local plate panels. As indicated by Eq. (4), \( \delta_{p0} \) generally depends on the panel slenderness and thickness. The PULS default value given as a function of a stiffener space only is not reasonable from this viewpoint, but it is used herein as one of the standard design values. For the stiffeners, the initial vertical deflection and the initial torsional deformation, having one half wave (\( m = 1 \)) over a span, Fig.2 (b), are assumed as

\[
W_{sp} = B_{sp} \sin \frac{\pi x}{a}, \quad \nu_{sp0} = C_{sp} \sin \frac{\pi x}{a}.
\]

Equation (3) gives the typical shape of initial imperfection of the stiffeners in ships. GL formulae for the ultimate strength of stiffeners are based on the same initial imperfection modes. The PULS initial imperfection mode that assumes the same number of half waves both in panels and in stiffeners is considered to be unrealistic for the stiffeners, when \( m \) is large.

3. Comparison between PULS and FEM

3.1 Rectangular plate panel under biaxial thrust

As the most fundamental case, the ultimate strength of a rectangular isolated plate panel simply-supported at all edges subjected to biaxial thrust is calculated first. In PULS, the program for unstiffened panel (U3) is used, and the simply-supported condition is specified at all edges through the input window shown in Fig.3.

The ultimate strength interaction relationships obtained for the two different plate thicknesses are shown in Fig.4. It is found that PULS overestimates the ultimate strength for the thick plates, particularly when the transverse thrust is dominant. The reason for this is probably as follows. For the case of thin plates, the ultimate strength is attained when the redistributed membrane stresses due to large deflection satisfy the yielding criteria at certain critical positions in the plate. On the other hand, for the case of thick plates, the ultimate strength is attained when the cross section is only partially yielded due to combined membrane and bending actions, i.e. before the full yielding due to membrane stresses is developed. Since PULS considers only membrane stresses in the yielding criteria, the judgment of yielding is delayed; resulting in the overestimate of the ultimate strength for the thick plates. When the transverse thrust is dominant, the rectangular plate fails predominantly in bending except for the region near the transverse edges, and the above error becomes more significant.
Another possible reason may be in that PULS basically deals with unstiffened or stiffened continuous panels whereas an isolated plate panel is considered in the present example. However, when free from stiffeners and lateral pressure, and particularly when the initial deflection is given by the elastic buckling mode, the buckling/ultimate strengths of a continuous unstiffened panel have to be the same as those of a simply-supported isolate plate. If some additional constraint is imposed in PULS model, it should be clarified.

Two kinds of initial panel deflection are considered in the FE analysis as shown in Fig.4. One is a buckling mode, Eq. (1), and the other a hungry-horse mode, Eq. (2). It is shown that the influence of the initial deflection mode on the ultimate strength of simply-supported isolated plates is relatively small. The influence for the case of continuous stiffened panel will be discussed in chapter 4.

3.2 Stiffened panel under longitudinal thrust

The ultimate strengths of a continuous stiffened panel with a tee-bar stiffener subjected to pure longitudinal thrust are calculated by PULS and FEM, taking the web depth as a variable parameter. The relationship between the calculated ultimate strengths and the stiffener heights is shown in Fig.5. The stiffener sizes under consideration are within the allowable range of cross-sectional geometries specified in the JTP rule. In PULS, the
initial imperfection based on the elastic buckling mode of a panel-stiffener combination is applied assuming the same number of half waves for the plate panel and the stiffener. In FEM, the initial deflection in a local buckling mode, Eq. (1), is assumed for the plate panels and that in an overall buckling mode, Eq. (3), for the stiffener. The corresponding results are given as a CASE-1 in Fig. 5.

As shown, the FE results show a falling strength tendency for the larger web heights, whereas PULS predicts the ultimate strength that is almost constant up to the stiffener height of 550 mm and even increases for the larger stiffener height. To explore the reason for this difference, the deformation at the ultimate strength is compared between PULS and FEM as shown in Fig. 6. In PULS, the short wave mode (m=6) resulting from the local initial imperfection mode develops in the plate panel and the stiffener, for the web height up to 550 mm, whereas in FEM the long wave mode (m=1) due to the lateral-torsional buckling develops in the stiffener. As is well known, the lateral-torsional buckling strength decreases with the increase in the web height. This is why the FE results show a monotonous decrease for the larger web heights. For the stiffener height of 650 mm, the local initial imperfection in PULS changes to the long wave mode (m=1), and the similar collapse modes are obtained by PULS and FEM. CASE-2 in Fig. 5 indicates the FEM results assuming the initial imperfection with a long wave mode (m=1) in the plate panel as well as in the stiffener. PULS assumes the similar initial deflection, but it predicts the larger ultimate strength than FEM. This is probably because the shape function based on the elastic buckling mode cannot fully simulate the reduction of effective cross section due to lateral-torsional buckling.

In PULS, the initial imperfection modes following a mode pattern consistent with elastic buckling modes of a panel-stiffener combination are systematically applied. But this does not necessarily mean the accurate estimate of the ultimate strength, particularly when the collapse is highly susceptible to the yielding as in the case of flexural or lateral-torsional buckling of the stiffener. In addition, the elastic buckling mode of a panel-stiffener combination with short waves, Figs. 6(a) and 6(c), seem to be unrealistic in the welded stiffened panels.

To validate the collapse mode obtained by FE analysis, an additional FE analysis is performed assuming a combination of the long (m=1) and short (m=5) wave initial imperfection modes for the stiffener. The maximum horizontal initial imperfection at the stiffener top is the same as in the Cases 1 and 2, i.e., 1/1000 of a stiffener span, and a half value is given to the long and short wave components each. The ultimate strengths obtained for the web height of 550 mm are shown in Fig. 5 as a Case-3, and the deformation mode and the spread of yielding in Fig. 7. As shown in Fig. 5, the ultimate strength of the Case 3 almost coincides with that of the Case 1. This means when the long and short wave imperfection modes coexist, the long wave mode develops at the final stage of
Fig.6 Comparison of collapse modes of stiffened panels having deep stiffeners between PULS and FEM \((r=15\text{mm})\)
collapse. PULS, and even FEM, that assumes only the short-wave initial imperfection mode may give an incorrect collapse mode for the stiffeners with large web height, leading to a significant overestimate of the ultimate strength. In addition, and perhaps more importantly in a sense, the collapse modes as shown in Figs. 6 (a) or 6(c) may give users an incorrect interpretation about collapse modes. It is thought that PULS can consider a combination of long and short wave deformation modes in its formulation [4]. This is expected to significantly improve the accuracy of PULS.

4. Comparison among PULS, GL formulae and FEM

The ultimate strength of a continuous stiffened panel subjected to biaxial thrust is calculated using PULS GL-formulae (hereafter called GL) and FEM for several combinations of plate thickness and stiffener type. The cross-sectional geometries of the stiffeners are given in Table 1. The stiffeners categorized in the same type have the same moment of inertia. The ultimate strength interaction relationships expressed in terms of the average longitudinal and transverse stresses, $\sigma_x$ and $\sigma_y$ respectively, are shown in Fig. 8.
In FEM, two types of initial panel deflection, a buckling mode (BL) and a hungry-horse mode (HH), are considered. The initial deflection of hungry-horse mode delays the occurrence of asymmetric modes of deflection due to buckling. The ultimate strength obtained for the initial deflection of hungry-horse mode is therefore larger than that for the initial deflection of buckling mode. The difference is larger for the larger plate thickness.

In GL, the ultimate strength of a single plate field (local plate panel) and that of a partial plate field (stiffener) are calculated separately, and the smallest value is taken as the ultimate strength of a stiffened panel. The panel-stiffener interaction effects including the increase of panel buckling strength due to the torsional stiffness of stiffeners and the fixation of stiffener’s torsional buckling caused by the attached plating are considered by specified correction factors. One typical characteristic of GL approach is a Poisson-effect correction made for a single plate field. If the working stresses are resulting from the FE analysis, it is regarded that they already include the stress share due to Poisson effect. Denoting the stresses which incorporate the Poisson effect by $\sigma^*_x$ and $\sigma^*_y$, the following reduced stresses $\sigma_x$ and $\sigma_y$ are used as input data for the ultimate strength formulae derived for the free-unloaded edge conditions.

$$\sigma_x = \sigma^*_x - \nu \sigma_y$$
$$\sigma_y = \sigma^*_y - \nu \sigma_x$$

The GL curves shown in Fig. 8 are the ultimate strength interaction relationship expressed in terms of $\sigma^*_x$ and $\sigma^*_y$ after Poisson-effect corrections. For comparison purpose, the original ultimate strength interaction relationship expressed in terms of $\sigma_x$ and $\sigma_y$ are shown in Fig. 9 by the thin lines. With the Poisson-effect corrections, the shape of ultimate strength interaction curves are drastically improved in the range of the medium stress ratios.

The major findings from Fig. 8 can be summarized as follows:

1. The ultimate strength interaction relationships for biaxial thrust condition obtained by PULS, GL and FEM are in good correlation with each other.
2. PULS and GL generally give conservative predictions of the ultimate strength compared to FEM, except for the loading condition of the item (4).
3. When the longitudinal thrust is dominant, GL gives significantly smaller ultimate strength than PULS and FEM. The estimate of the ultimate strength for the partial plate field (stiffener) under predominantly longitudinal thrust should be improved.
4. When the transverse thrust is dominant, both PULS and GL significantly overestimate the ultimate strength, particularly for the thick plates. The possible reasons have been discussed for PULS in 3.1, but are not clear for GL.
5. When the longitudinal thrust is dominant, both PULS and GL underestimate the ultimate strength of a
thick plate. When compared with the FEM results obtained for the initial panel deflection of the hungry-horse mode, the difference is not negligible. This indicates that the significant strength reserve may exist in the present design scantling.

It is to be noted here that the Poisson-effect correction in GL is based on the assumption that the unloaded edges under uniaxial-thrust condition are non-displaceable in their perpendicular directions. Such a constraint is not necessary the true in the case of a single-plate field in the real continuous structures. In addition, the Poisson-effect appears in different ways in the pre-buckling, pre-collapse and post-collapse regimes. The correction based on the Poisson ratio only, Eq. (5), is therefore not physically reasonable. Furthermore, the Poisson-effect correction is specified as 'may condition', i.e. it is not strictly required. The guideline for the application of Poisson-effect correction should be more clarified as well as its background.

It is expected that further efforts will be made regarding the above items for the improvement and future harmonization of JTP and JBP methods.

5. Conclusions

The following conclusions can be drawn from the present study:

(1) The ultimate strength interaction relationships obtained by PULS, GL and FEM are in good correlation with each other. PULS and GL generally fall on the conservative side, compared to FEM.

(2) PULS and GL tend to overestimate the ultimate strength of the thick plates under predominantly transverse thrust, whereas tend to underestimate that under predominantly longitudinal thrust.

(3) PULS cannot correctly represent the lateral-torsional buckling behavior of stiffeners with large web height, resulting in the overestimation of the ultimate strength.

(4) The Poisson-effect correction in GL has a significant influence on the predicted ultimate strength. The physical background and the guideline for its application are expected to be more clarified.

References


Fig. 8 Ultimate strength interaction relationships of stiffened plate under bi-axial thrust (Comparison among PULS, GL and FEM)
Fig. 9 Ultimate strength interaction relationships of stiffened plate under bi-axial thrust
(Comparison of GLs with and without considering Poisson-effect correction)